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A Theoretical Investigation of Transformer Design

Electrical Engineering

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A THEORETICAL INVESTIGATION OF TRANSFORMER DESIGN

BY

JOHN ROBERT COLVILLE
AND
EDWIN LEWIS CONNELL

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

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May 31, 1912

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

JOHN ROBERT COLVILLE AND EDWIN LEWIS CONNELL

ENTITLED A THEORETICAL INVESTIGATION OF TRANSFORMER DESIGN

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

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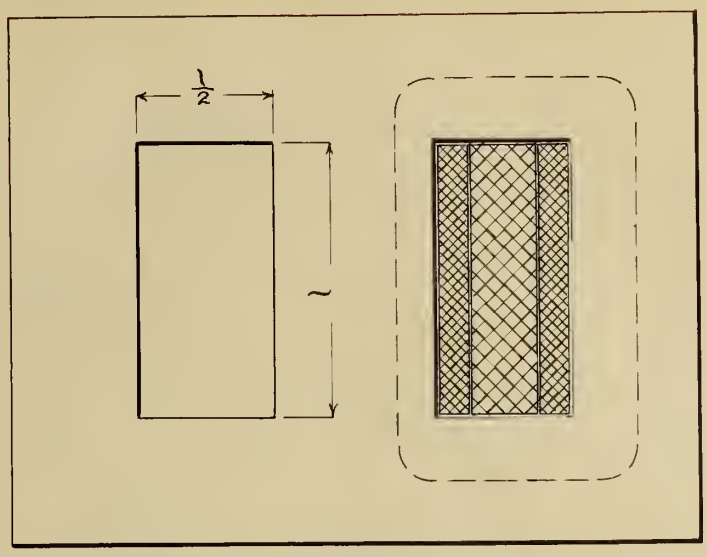
INTRODUCTION.

In the investigation of transformer design from a theoretical standpoint, there are several considerations which must be fixed before any definite progress can be made. Each class of service has its particular requirements. For distribution in lighting systems high all-day efficiency and good regulation are essential, and sixty-cycle supply is almost universally used. Having these requirements to fulfill the designer must choose the type of transformer and strive for the best results obtainable at a reasonable cost.

The first step in the design of a transformer is the choice of the number of turns, and, since the designer is unable to tell whether he has made a correct assumption until almost through with the calculations, it is very desirable that a simple formula be at hand which will furnish a starting point that may be relied upon to give a good transformer. The determination of such a formula is the purpose of this thesis.



MECHANICAL DESIGN

30 K.V.A.–Scale $\frac{1}{4}'' = 1''$

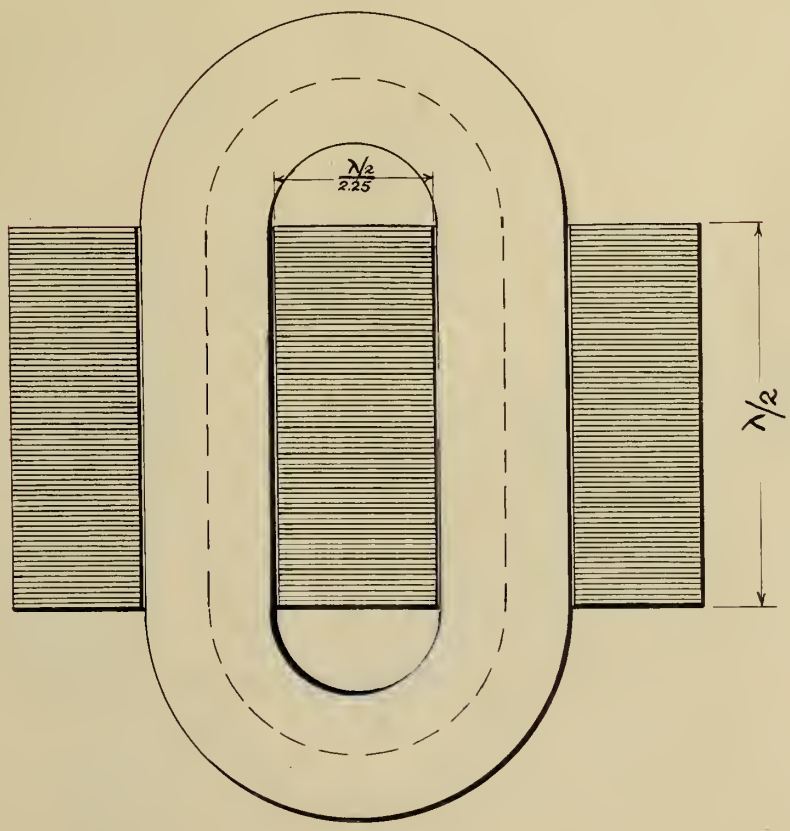
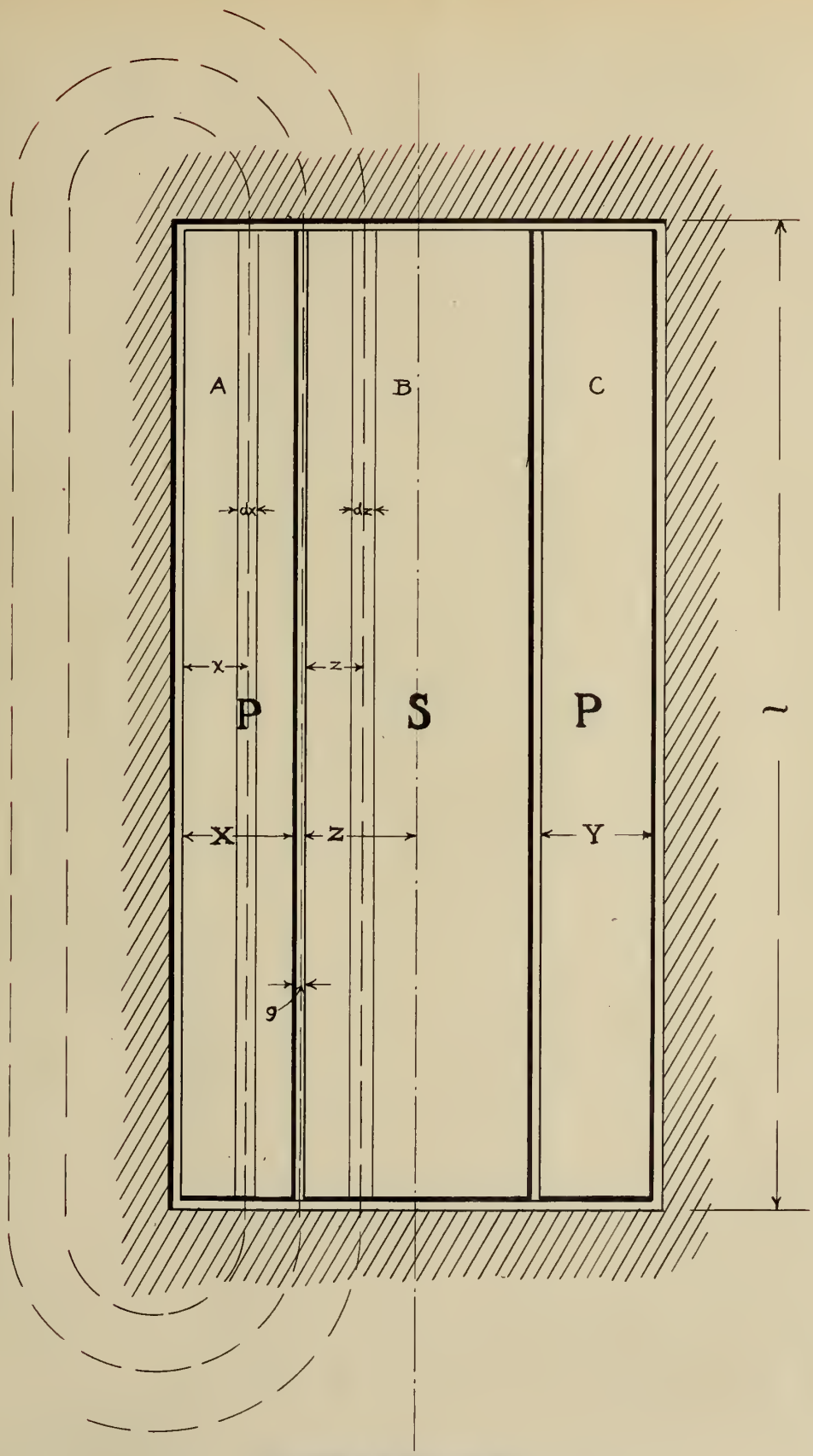


Fig. 1

EXPLANATION OF METHOD.

In the determination of a formula to give the proper number of volts per turn upon which to base transformer design, it is absurd to suppose that one could be found which would hold accurately for all styles and types in use at the present time. It was decided that this investigation be limited to shell type transformers up to 50 K. V. A. capacity for distribution on a 60-cycle lighting system.

For this class of service the transformer will generally be only lightly loaded the greater part of the twenty four hours. The iron losses are practically constant at all loads, and the copper losses vary as the square of the current, consequently the minimum loss will be obtained if the transformer is designed for greatest efficiency at the root mean square of the all-day load. However, the load-factor may be so low that such a design is not practicable owing to the excessive heating at the maximum loads. Also the design to give least total loss is not necessarily the most economical. The central station equipment must include boilers, prime movers, and generators of capacity sufficient to take care of the maximum load demanded by the system and during light loads the boilers must be kept in reserve for these peak-loads. With the boilers on light load the cost of the losses in the transformers of the system is very small, whereas, during the peak-load periods excessive losses would require extra equipment and the cost of the power to supply these losses might thus amount to much more than the cost of losses throughout the light-load period. For this reason these transformers have been designed for maximum efficiency at full rated load.



Leakage Flux

Fig. 2

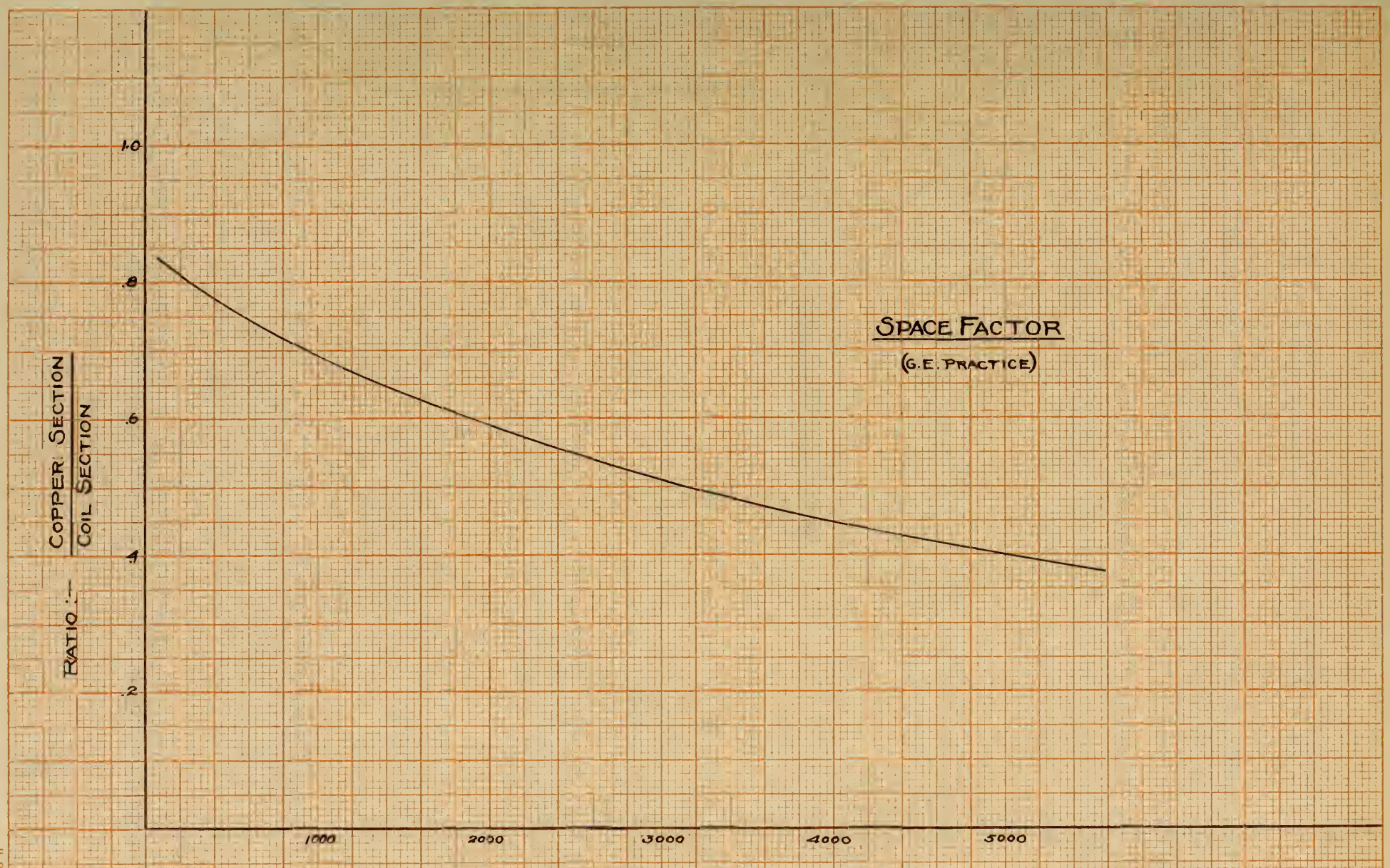

SPACE FACTOR
(G.E. PRACTICE)
RATIO :— COPPER SECTION / COIL SECTION
1.0
.8
.6
.4
.2
1000
2000
3000
4000
5000
— VOLTS —

Fig 3

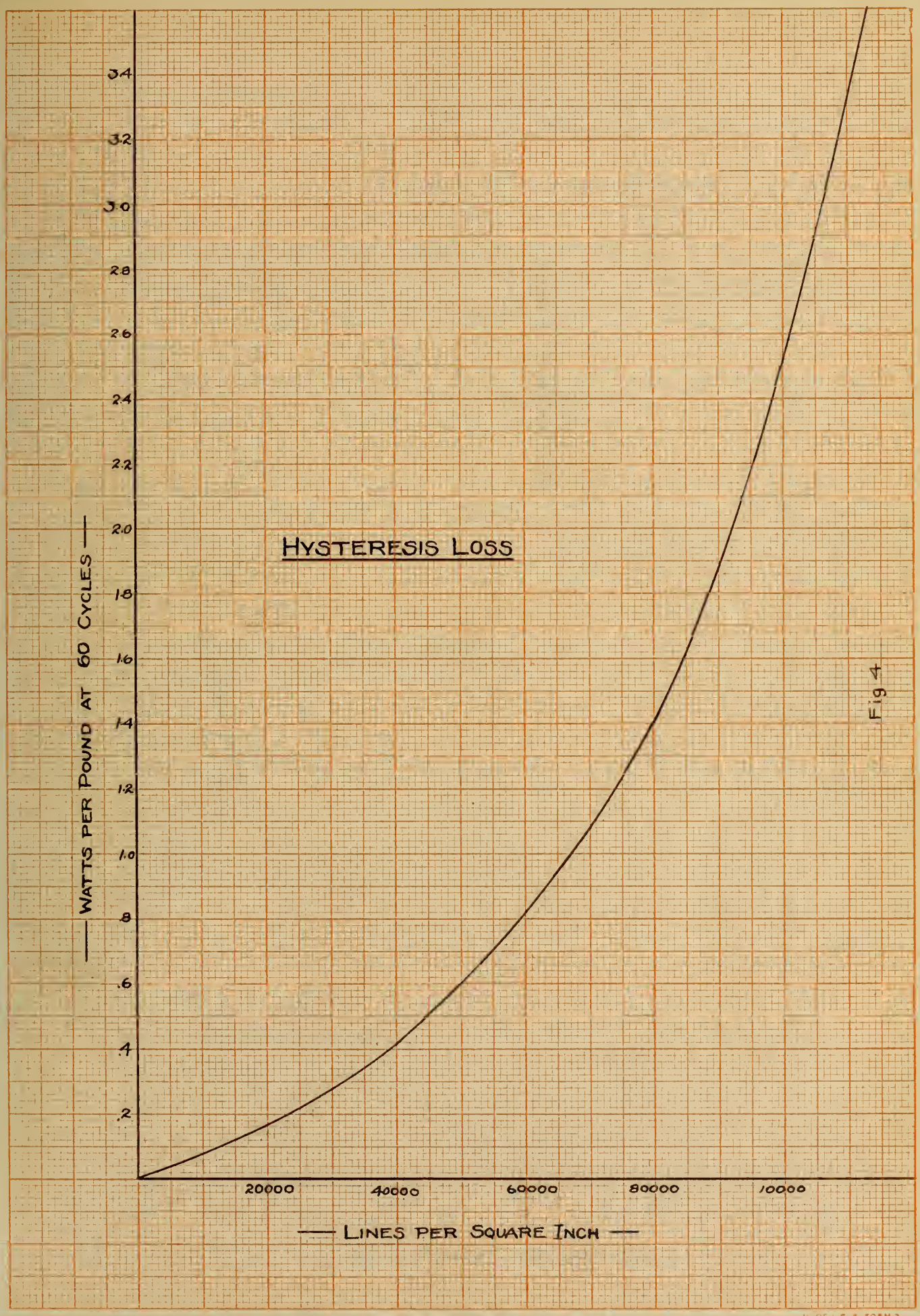


Fig 4

U. OF I. B. S. FORM 3

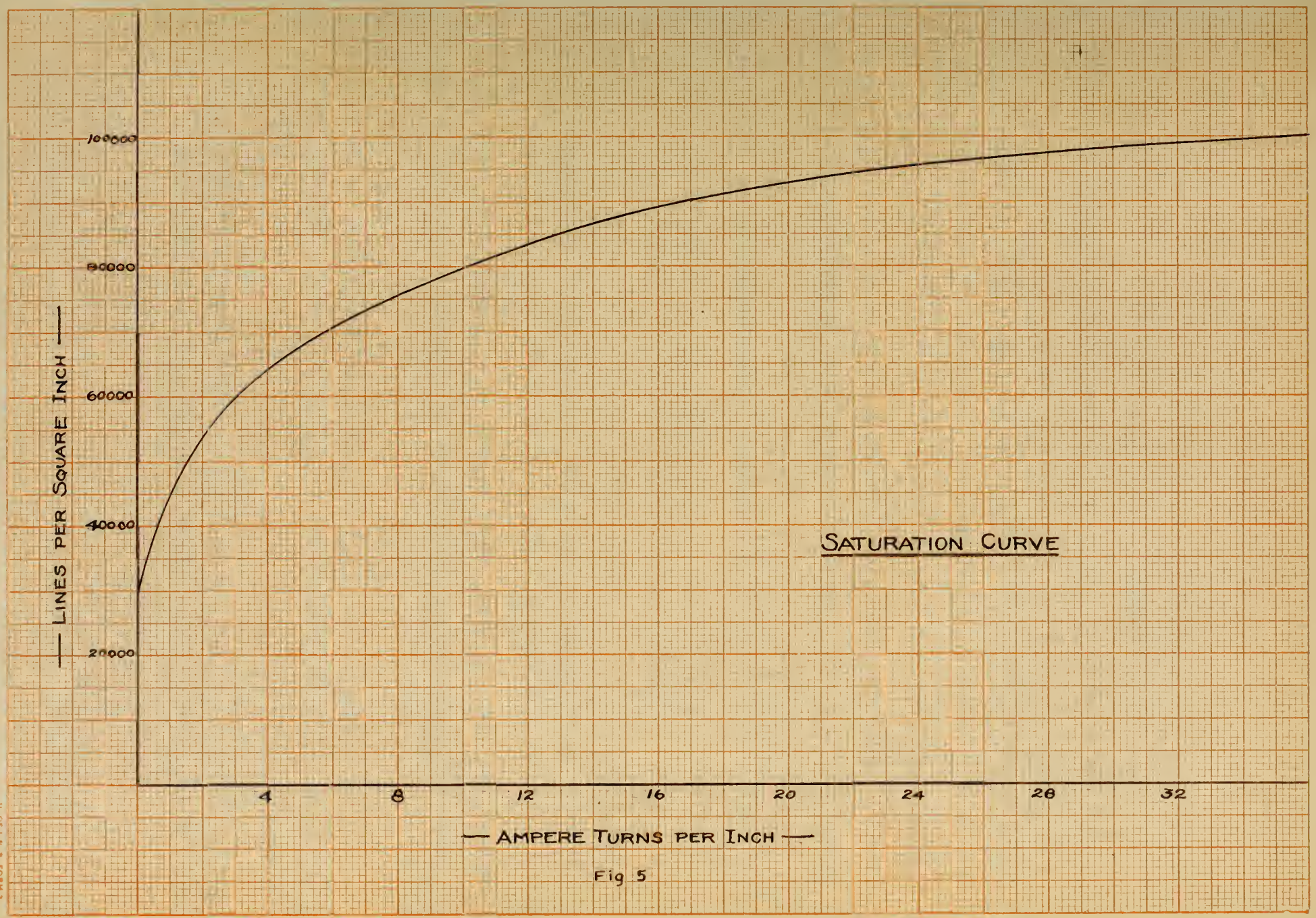


Fig 5

U. OF I. E. S. FORM 3

Since the calculations were made for the comparison of the electrical characteristics, the mechanical design has been systematized in many ways. The general proportions of the magnetic and copper circuits were made uniform by the adoption of a core-section with dimensions 2.25 : 1, and a window with dimensions 2 : 1. These relations were chosen after investigating the effect of shape on the cost and reactance. A scaled drawing of the 30 K. V. A. size is included herewith, Fig. 1 . The cost of material has been calculated on the basis of iron at four cents and copper sixteen cents per pound. The area of the window is determined by the ratio of the copper section to the section of the finished coil. The space-factor curve, Fig. 3 represents standard practice in transformer insulation up to about 5,000 volts. For the low tension side a space factor of .8 was adopted. Between coils, and between coils and iron, a .06 in. press-board insulation was allowed. The effective area of the laminated iron section was taken as .9 the actual area. A good grade of iron was assumed; the curve shown in Fig. 4 shows hysteresis loss in watts per pound at 60 cycles for a large range of densitities, and Fig. 5 gives the permeability in terms of ampere turns per inch for the same range of densities. The resistance of the conductors was calculated on the basis that

$$R = \rho \frac{1 \text{ ft.}}{\text{a sq. in.}}$$

$$\rho = .00000947$$

The cross section of the copper conductor was determined by allowing a density of 1,000 amperes per square inch.

The calculation of transformer reactance is necessarily only an approx-

imation. The method of derivation of the formula used is here given. The diagram is shown in Fig. 2 and all dimensions are in centimeters.

The leakage-flux which causes the reactance is divided into three separate and distinct fluxes, the leakage-flux of the primary ϕ', the leakage-flux of the secondary ϕ'', and the leakage-flux which passes through the insulation spaces between coils, ϕ'''.

First, consider the coil A. Let $d\phi'$ represent the flux through the differential area of width dx. The mmf producing this flux is

$$4\pi N_a i_a \times \frac{x}{X}$$

The reluctance of the path as shown in Fig. 1 and Fig. 2 is $\frac{l}{\lambda\, dx}$

Then since

$$d\phi = \frac{mmf}{R}$$

$$d\phi'_a = \frac{4\pi N_a i_a x \lambda\, dx}{Xl}$$

This flux does not encircle the entire number of turns, but only $\frac{x}{X}$ part of them. Therefore it may be assumed that $\frac{x}{X}$ of the entire flux cuts all the turns in Coil A.

Therefore

$$d\phi'_a = \frac{4\pi N_a i_a \lambda}{lX^2} \int_0^X x^2 dx$$

$$= \frac{4\pi}{3} \frac{N_a i_a \lambda X}{l}$$

In an exactly similar manner the leakage-flux in C

$$\phi'_c = \frac{4\pi N_c i_c \lambda Y}{3\ l}$$

Then the total primary leakage-flux is

$$\phi' = \phi'_a + \phi'_c = \frac{8\pi N_a i_a \lambda X}{3\ l}$$

since $X = Y$, $N_a = N_c$ and $i_a = i_c$

But N_a is only half of the total primary turns. Calling N' total primary turns, then:

$$\phi' = \frac{4\pi N' i_p \lambda X}{3\ 1}$$

Referring to the flux-diagram, Fig. 2, it is seen that half the leakage-flux of the secondary cuts half the primary turns and the other half cuts the other half of the primary. This may be considered as all of the secondary leakage-flux cutting half the primary or, for the sake of convenience, as half of the secondary flux cutting all of the primary. The equation then becomes:

$$d\phi'' = \frac{4\pi N'' i_s Z^2 \lambda\, dZ}{Z^2 l}$$

$$\phi'' = \frac{4\pi N'' i_s \lambda Z}{3\ 1}$$

The last leakage-flux to be considered is that through the insulation spaces g. This flux is set up by the total mmf of either the primary or secondary, it makes no difference which, as the two are equal. Half of the mmf may be considered as spent in each insulation-space between the coils, or all may be taken as sending the flux through one space g. Considering this

the latter way

$$mmf = 4\pi N' i_p$$

$$R = \frac{\lambda g}{l}$$

$$\phi''' = \frac{4\pi N' i_p \, g \lambda}{l}$$

Adding ϕ', ϕ'', ϕ'''

$$\phi_L = \frac{4\pi N' i_p \lambda}{l}\left[\frac{X}{3} + \frac{Z}{3} + g\right]$$

Now

$$L = N\frac{\phi}{i}$$

$$L = \frac{4\pi (N')^2 \lambda}{l}\left[\frac{X}{3} + \frac{Z}{3} + g\right] \text{ henries}$$

To this value of L must be added some quantity to take care of the leakage-flux in the end turns or those turns which project beyond the core.

Therefore the final form of the equation becomes

$$L = \frac{4\pi (N')^2 \lambda}{l}\left[\frac{X}{3} + \frac{Z}{3} + g\right] \times 1.15 \text{ henries}$$

Since the dimensions of the core-section determine the length of the mean turn, it is evident that by increasing the density in this part of the circuit the cost, copper loss and also the reactance may be reduced. However, the iron loss and especially the magnetizing current will be increased at the same time. To determine what density is best for this part of the circuit, 10 and 30 K. V. A. transformers were designed for four different densities, 60,000, 75,000, 90,000 and 105,000 lines per square inch, the density in the yoke being kept constant at 60,000. Having no basis upon which to choose the proper number of turns, different values of volts per turn were used until reasonable results were obtained. The curves, Fig. 6 , 7 , 8, 9 , 10 , 11, show the results, and if the better values of volts per turn be examined it will be found that the density in the core can be increased to 90 000 without requir-

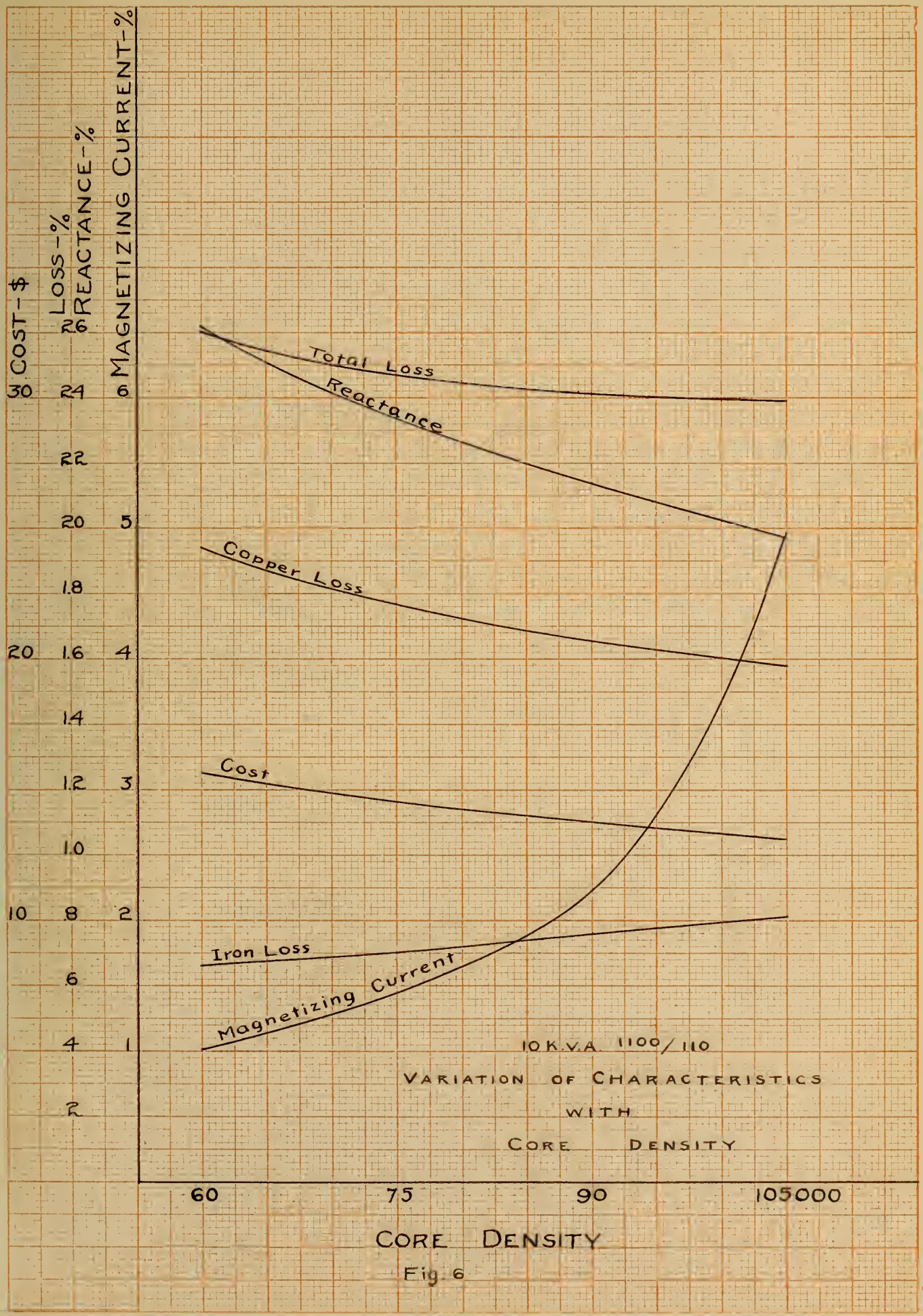

COST-$
LOSS-%
REACTANCE-%
MAGNETIZING CURRENT-%
30
20
10
26
24
22
20
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
6
5
4
3
2
1
Total Loss
Reactance
Copper Loss
Cost
Iron Loss
Magnetizing Current
10 K.V.A. 1100/110
VARIATION OF CHARACTERISTICS
WITH
CORE DENSITY
60
75
90
105000
CORE DENSITY

Fig. 6

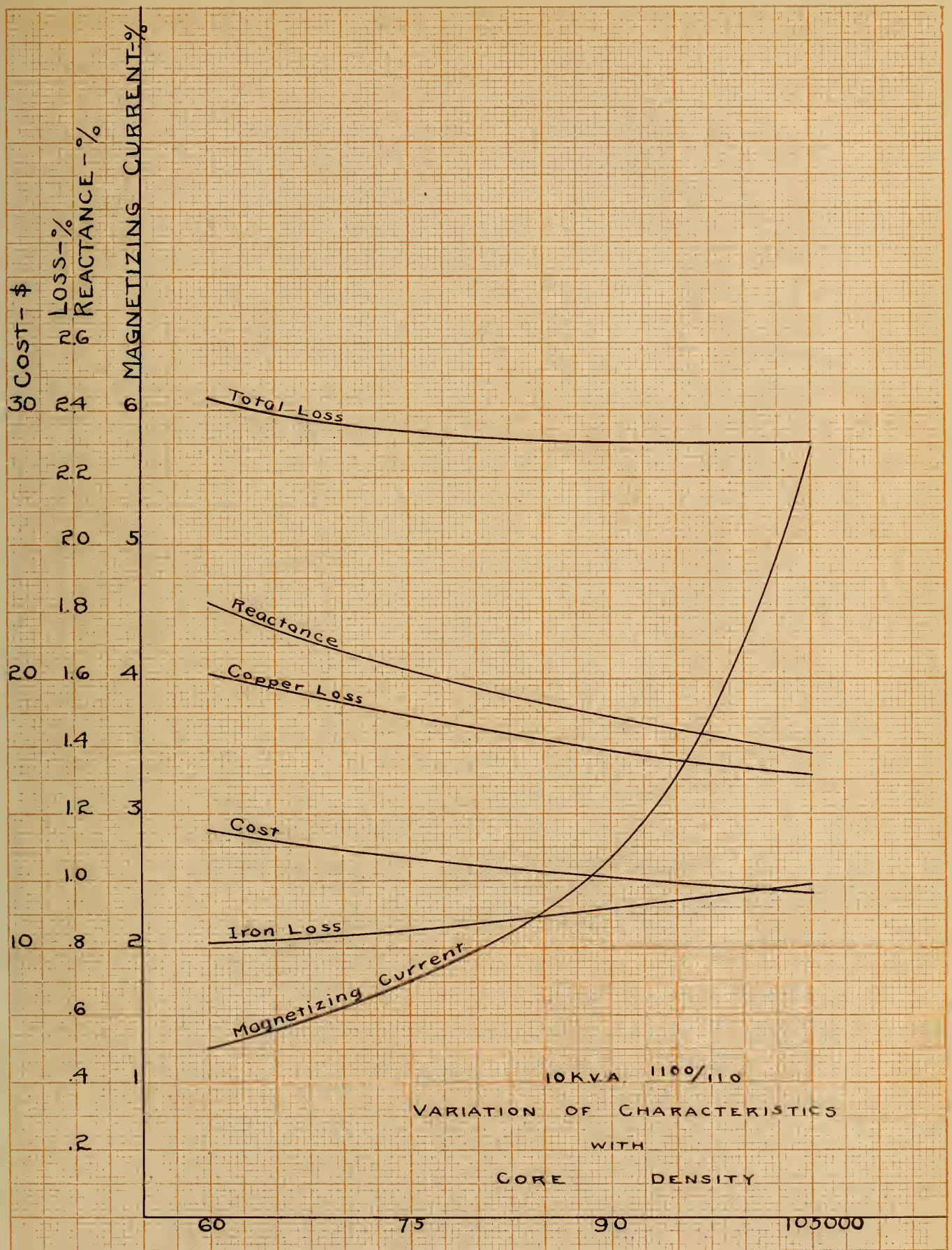


Fig 7

U. OF . S. S. FORM 3

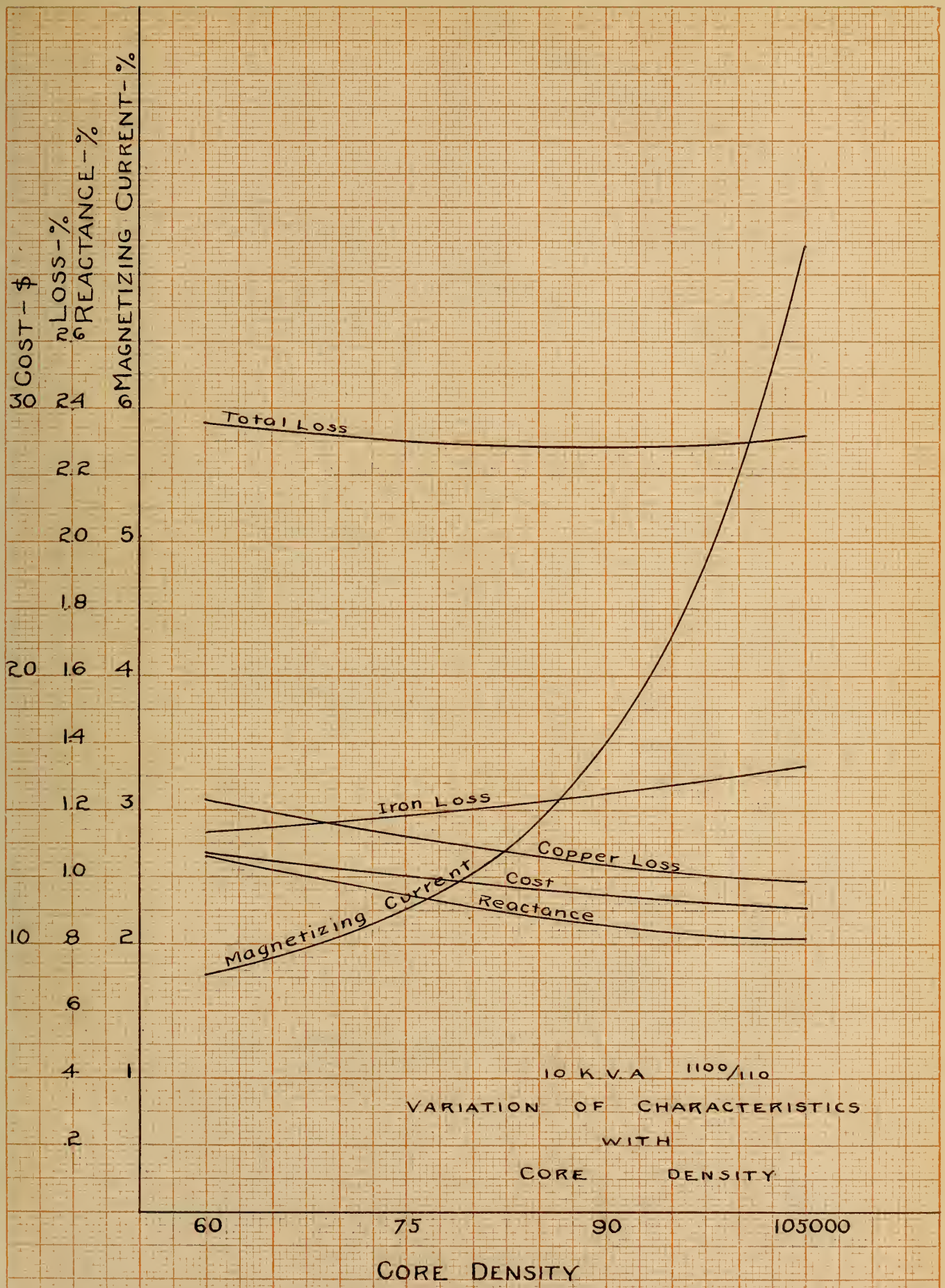


Fig 8

U. OF I. S. S. FORM 3

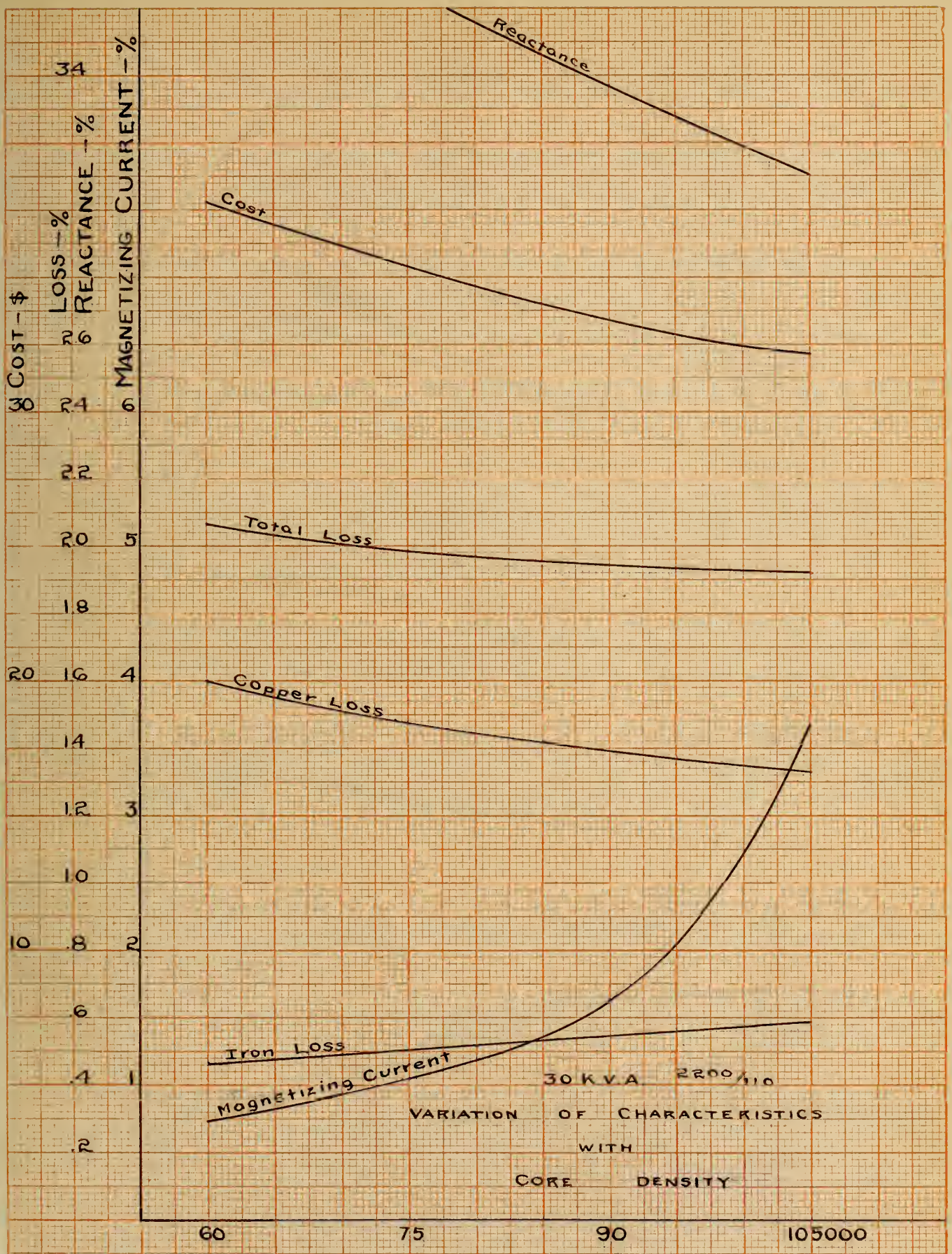


Fig 9

U. OF I. E. E. FORM 3

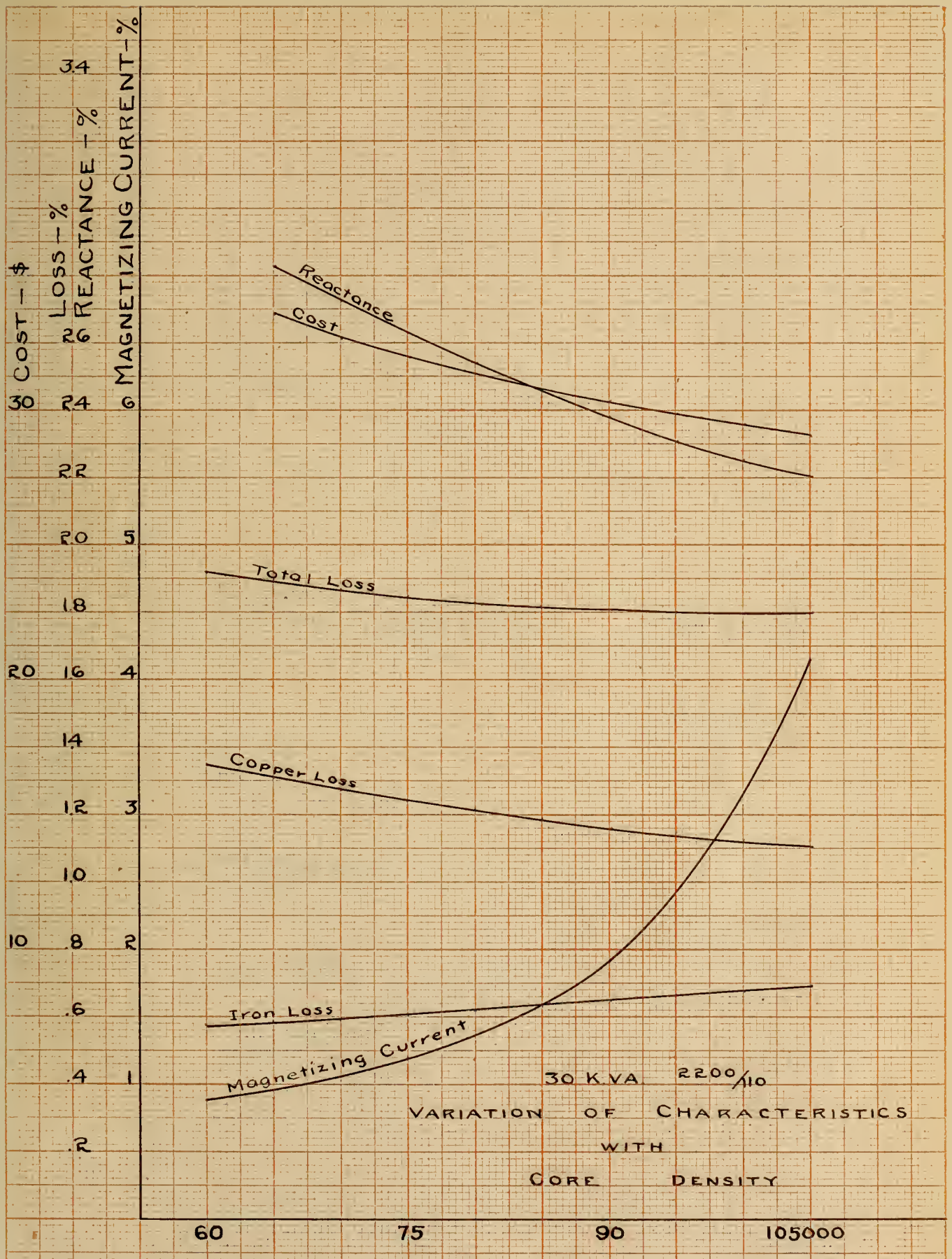

Cost — $
Loss — %
Reactance — %
Magnetizing Current — %
3.4
26
30
24
6
22
20
5
1.8
20
16
4
14
12
3
1.0
10
.8
2
.6
.4
1
.2
Reactance
Cost
Total Loss
Copper Loss
Iron Loss
Magnetizing Current
30 K.VA 2200/110
VARIATION OF CHARACTERISTICS
WITH
CORE DENSITY
60
75
90
105000
CORE DENSITY

Fig. 10

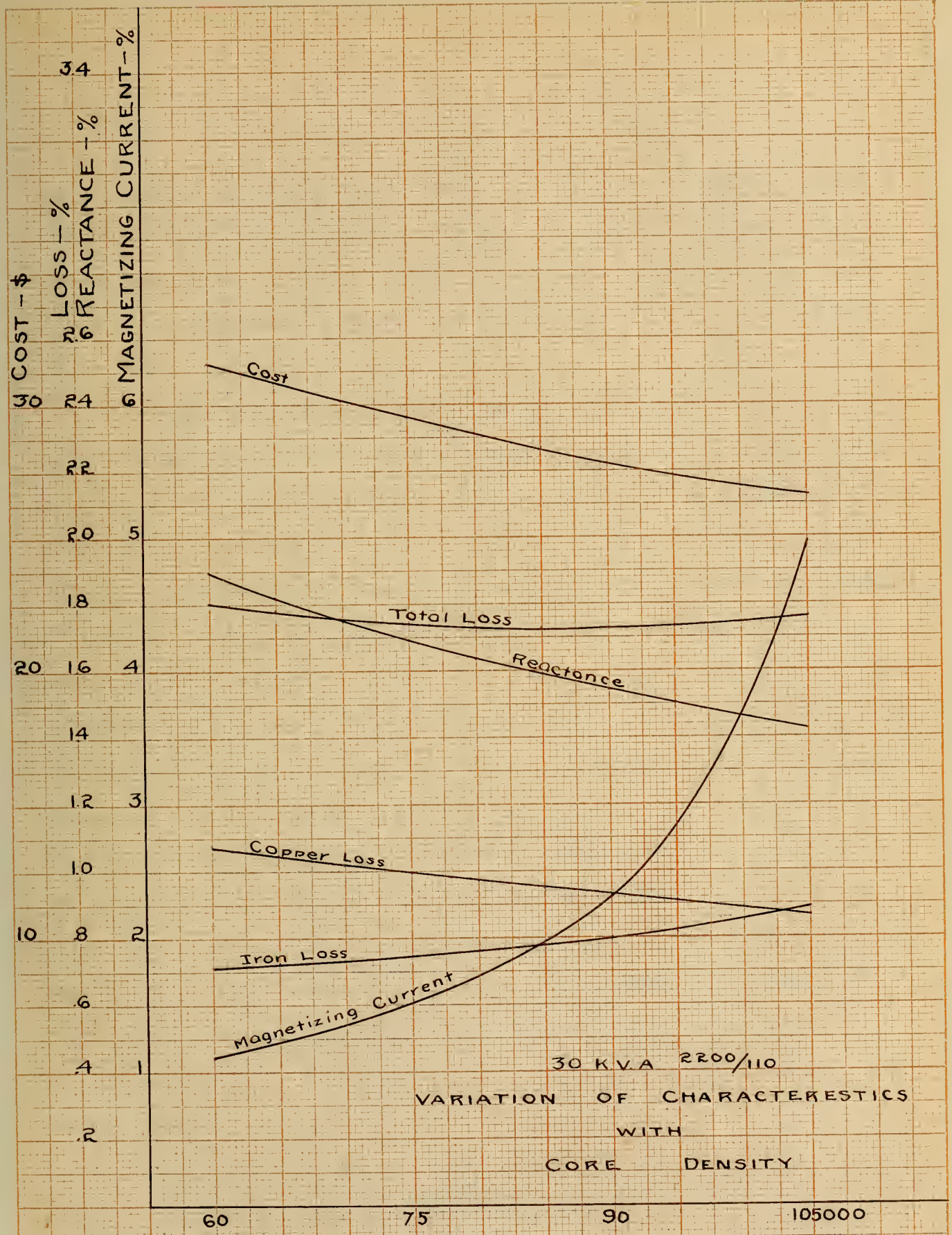

COST - $
LOSS - %
REACTANCE - %
MAGNETIZING CURRENT - %
3.4
2.6
30
2.4
6
2.2
2.0
5
1.8
20
1.6
4
1.4
1.2
3
1.0
10
.8
2
.6
.4
1
.2
Cost
Total Loss
Reactance
Copper Loss
Iron Loss
Magnetizing Current
30 KVA 2200/110
VARIATION OF CHARACTERESTICS
WITH
CORE DENSITY
60
75
90
105000
CORE DENSITY

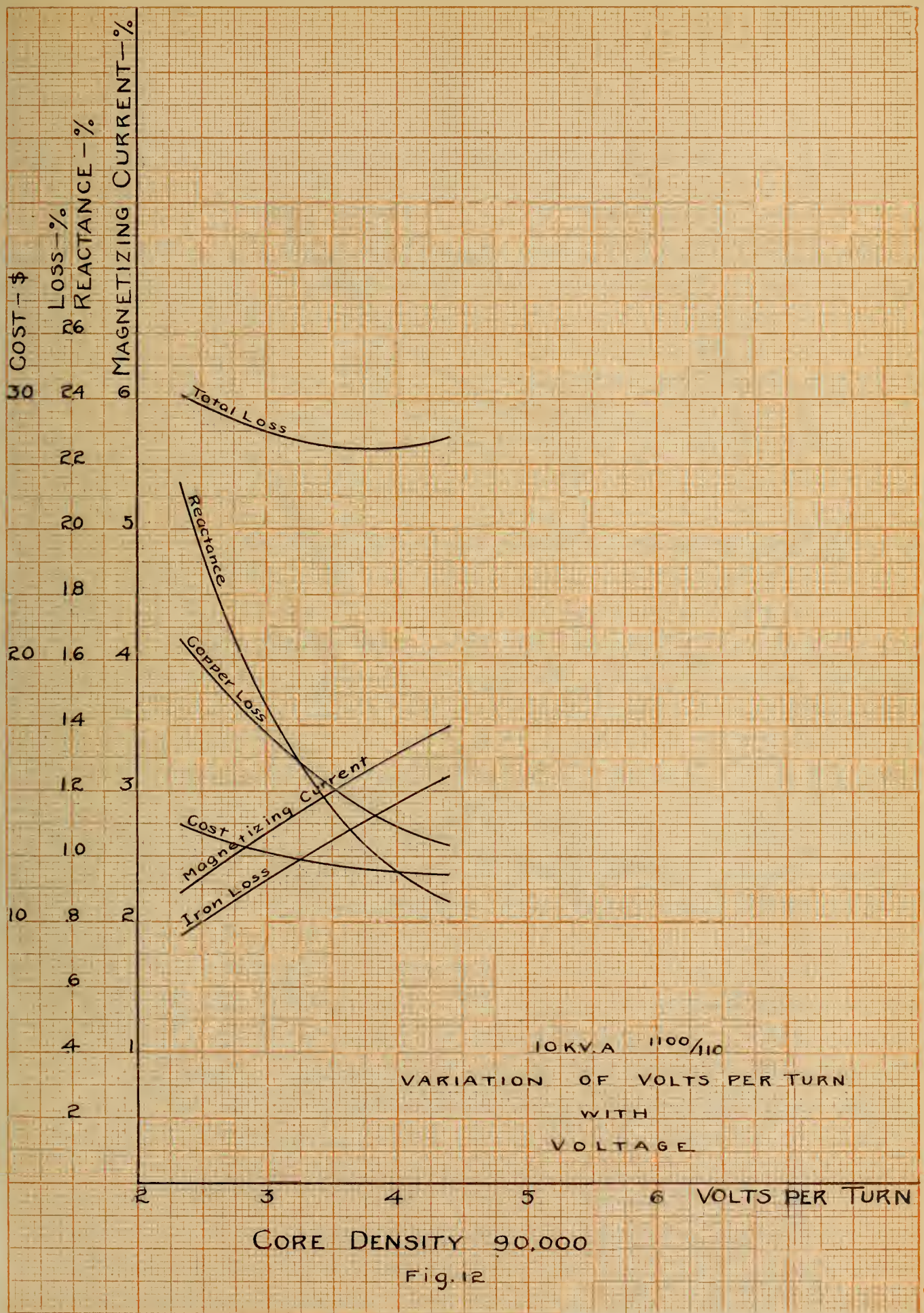
Fig. 11

ing an objectionable magnetizing current. There will, of course, be large currents produced upon closing the switch at the zero point of the emf wave due to transients, but this is more or less true of all transformers. The effect on the iron loss is slight, the saving otherwise made easily outweighs it.

With the core-density 90,000 and yoke 60,000, calculations were then made to ascertain the proper number of volts per turn, and to show the variation of volts per turn with the voltage. The 10 and 30 K. V. A. transformers were designed for four values of volts per turn, and curves plotted for cost of material, iron loss, copper loss, total loss, reactance and magnetizing current. The crossing point of the two loss curves is taken as the design condition desired. These two sets of curves, Fig. 12, 13, 14 were calculated for two voltages, 1100 and 3300 for the 10 K. V. A. and 1100 and 4400 for the 30 K. V. A. The results show that the volts per turn are almost independent of the voltage. Using a 20 : 1 ratio, calculations were then made for four different values of volts per turn for each of five different capacities, viz., 10, 20, 30, 40, and 50 K. V. A. The results are given graphically in curves, Fig. 15, 16, 17, 18, and 19. With the volts per turn for equal copper and iron losses as taken from these curves, i. e., from the point of intersection of iron loss-curve and copper loss-curve, a curve was plotted between K. V. A. capacity and volts per turn, see Fig. 20. The equation of this curve is the formula sought.

The curve is a parabola represented by the following simple equation,

$$\frac{V}{T} = 1.22 \sqrt{K.\ V.\ A.}$$



CORE DENSITY 90,000

Fig. 12

U. OF I. E. S. FORM 3

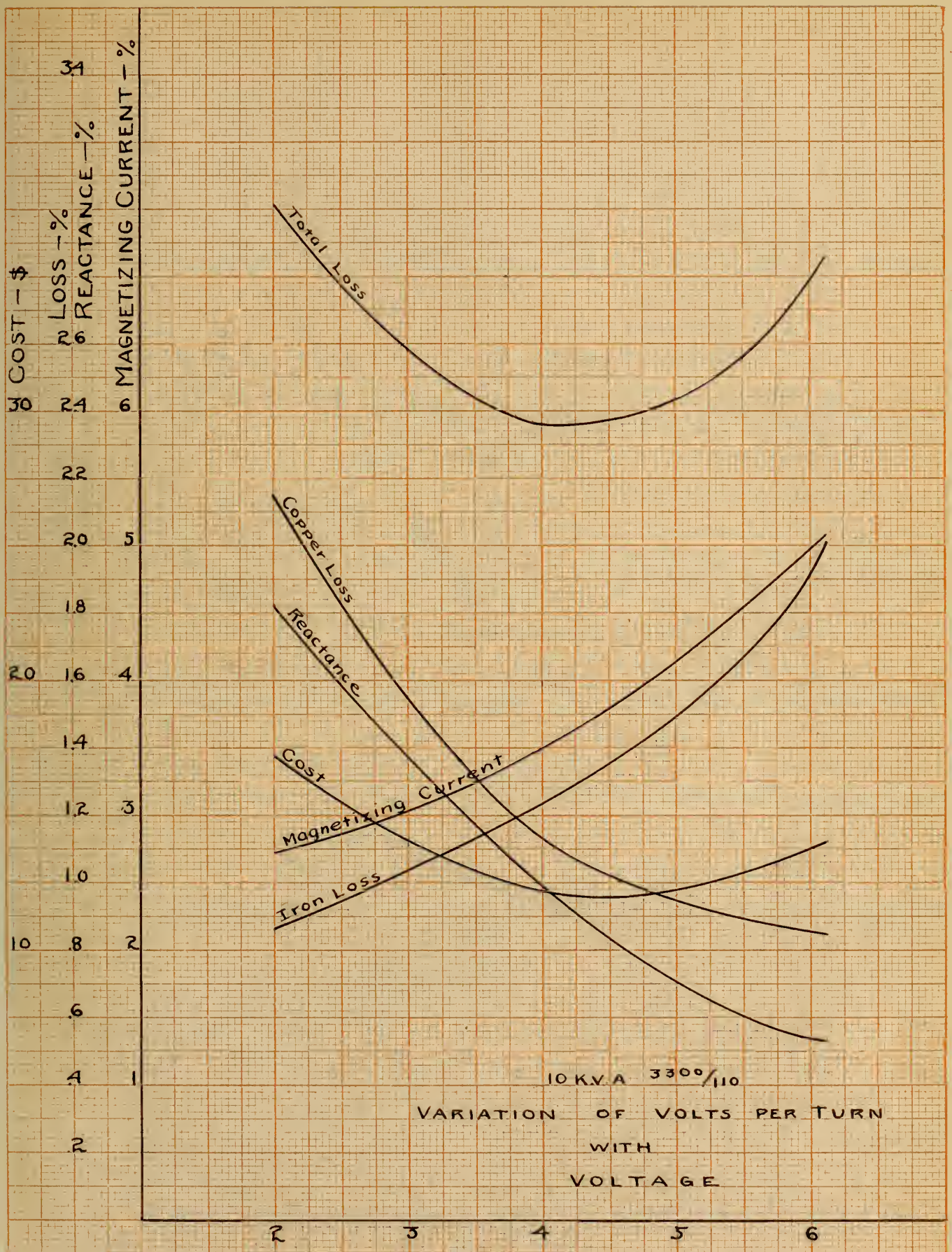


Fig. 13

U. OF I. & S. FORM 3

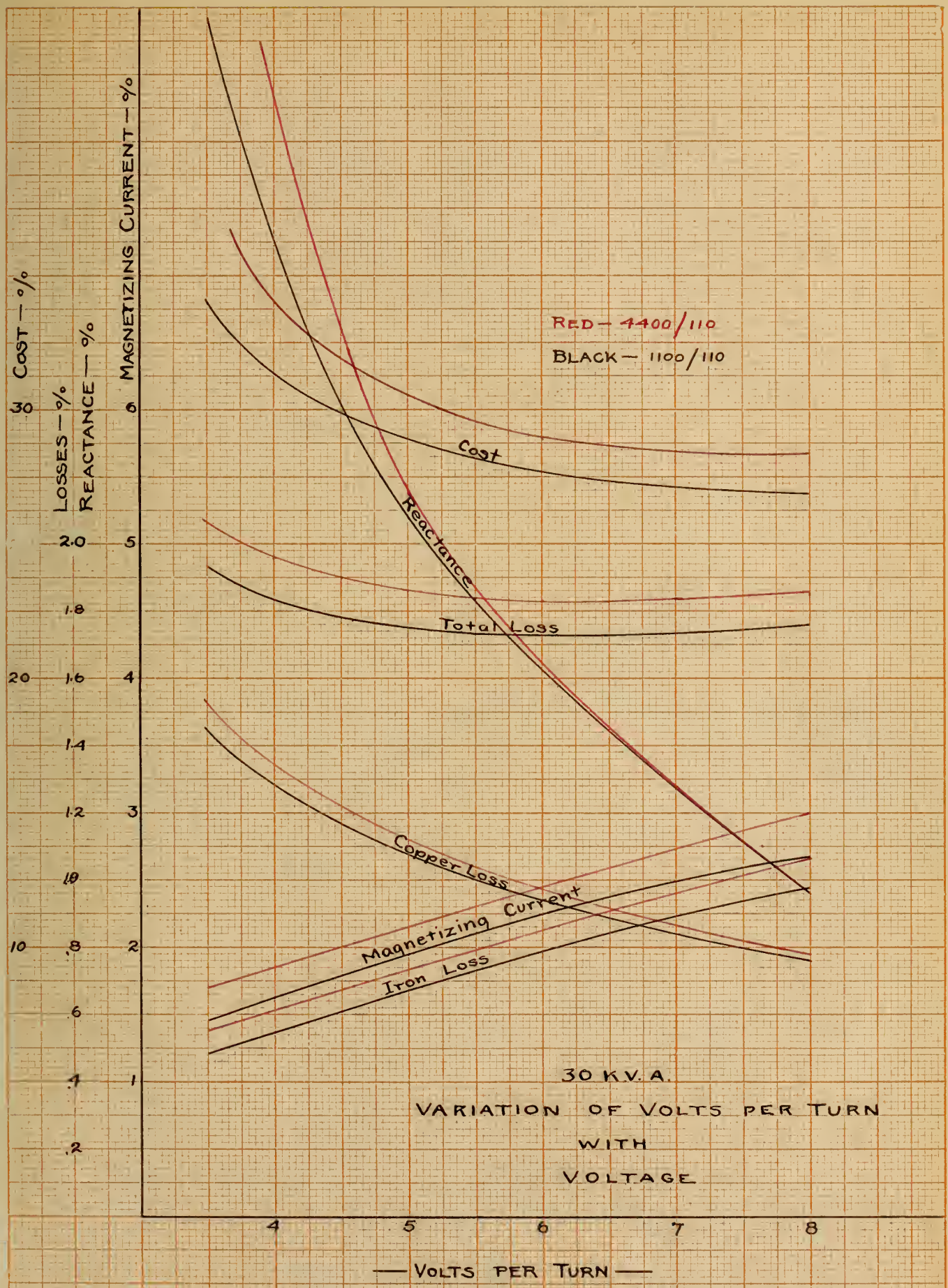


Fig. 14

U. OF I. E. S. FORM 3

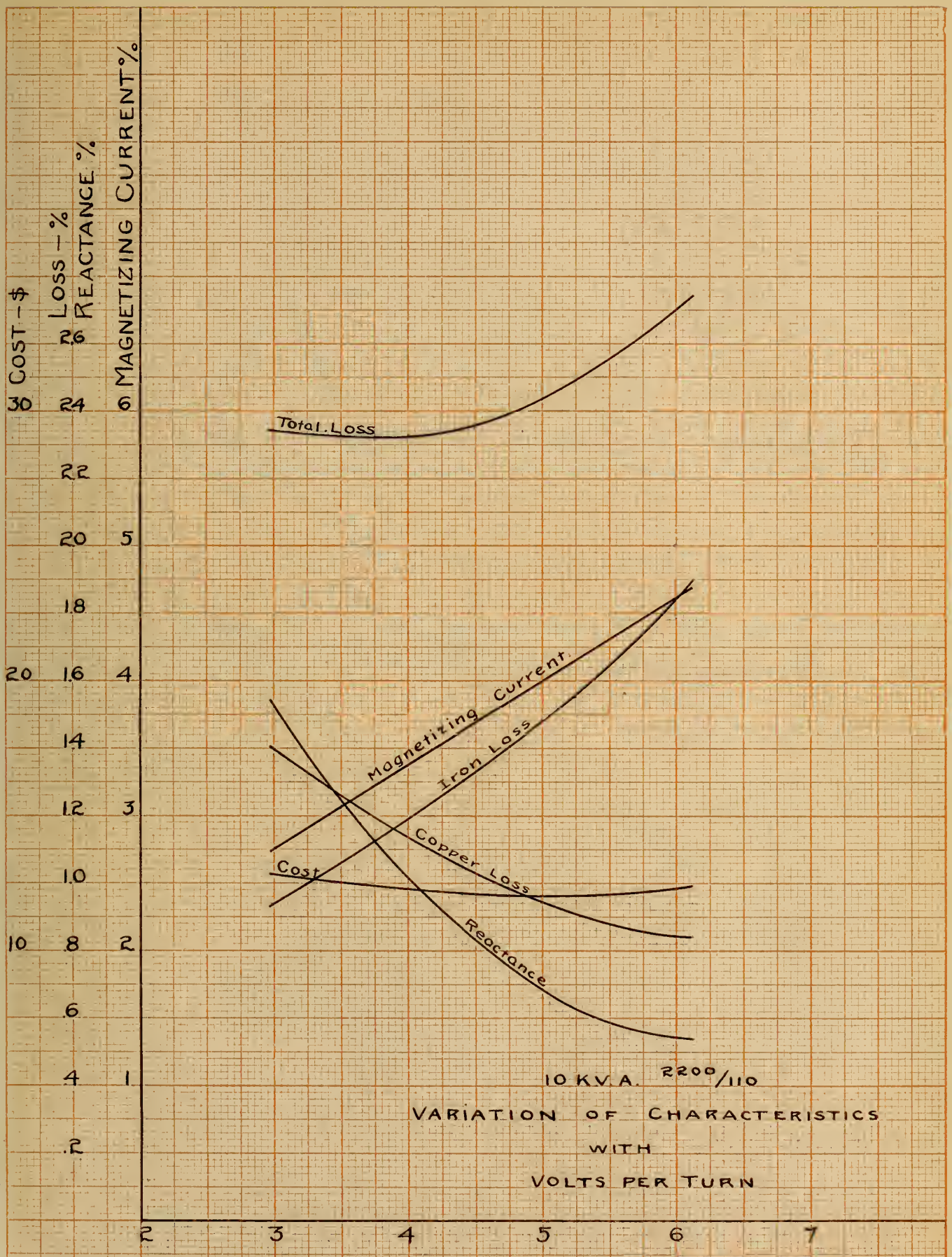


Fig. 15

U. OF I. E. S. FORM 3

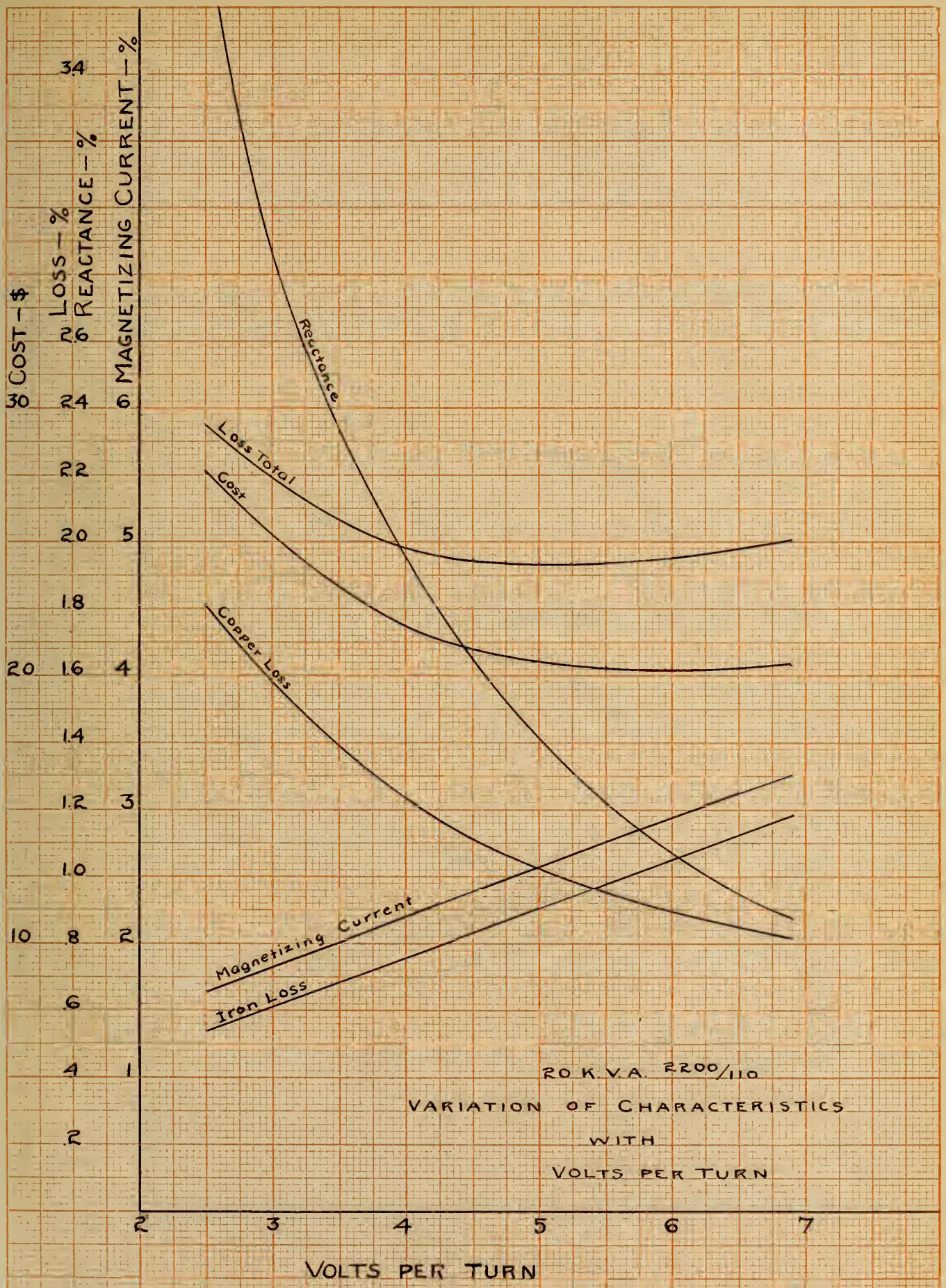


Fig. 16

U. OF I. E. E. FORM 3

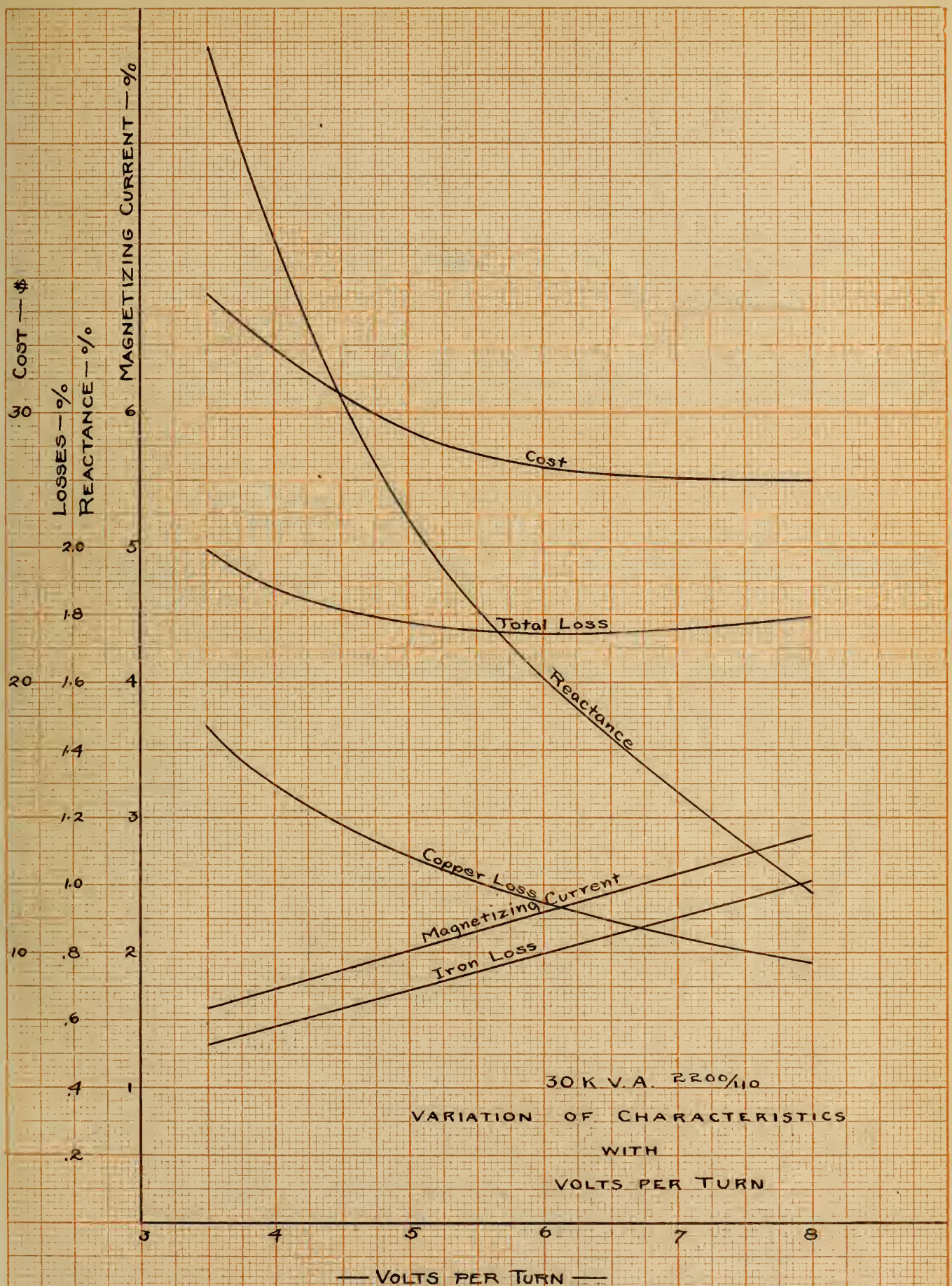

Cost — $
Losses — %
Reactance — %
Magnetizing Current — %
30
20
10
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
6
5
4
3
2
1
Cost
Total Loss
Reactance
Copper Loss
Current
Magnetizing
Iron Loss
30 K.V.A. 2200/110
VARIATION OF CHARACTERISTICS
WITH
VOLTS PER TURN
3
4
5
6
7
8
— Volts per Turn —

Fig. 17

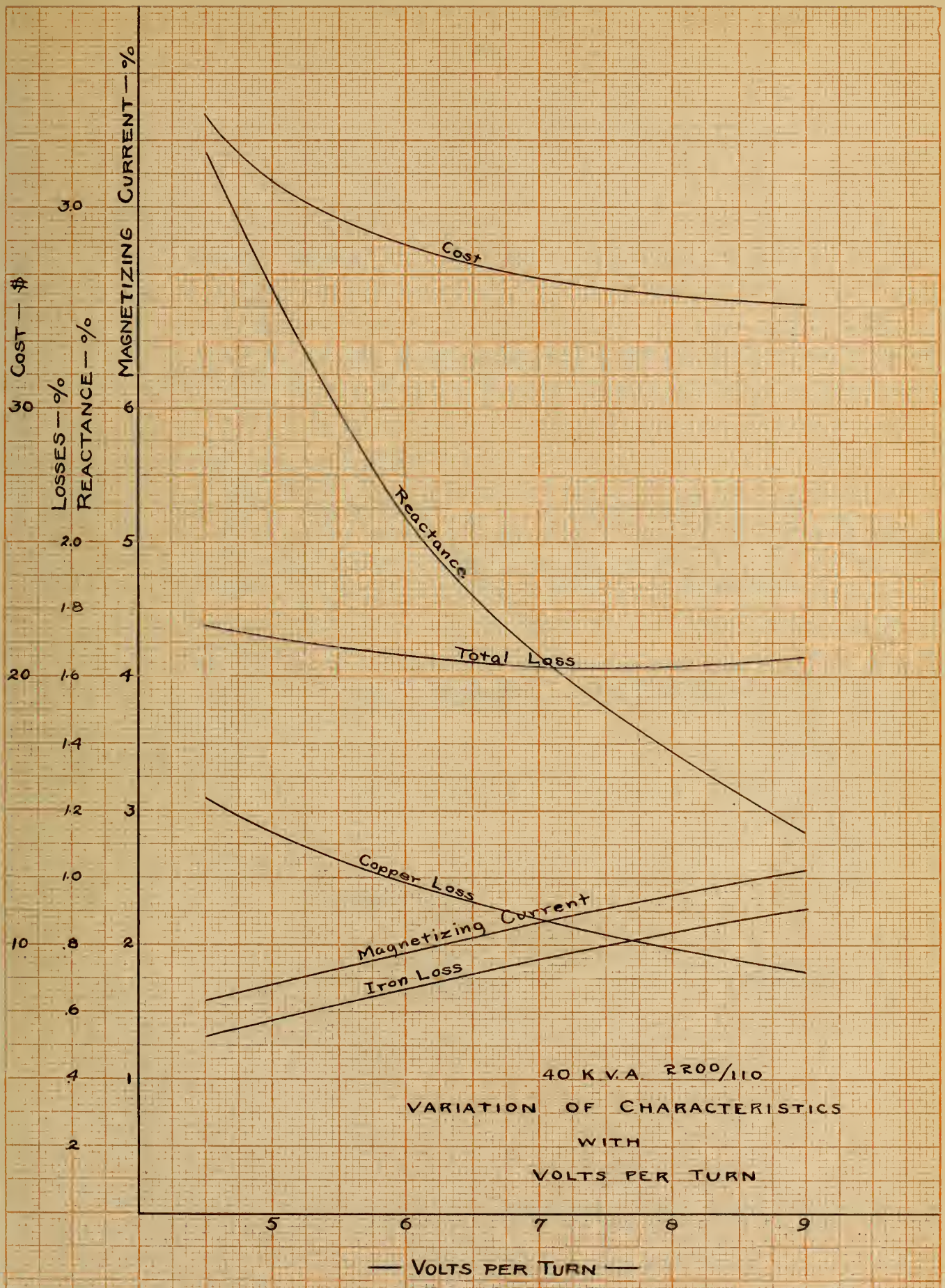


Fig 18

U. OF I. E. E. FORM 3

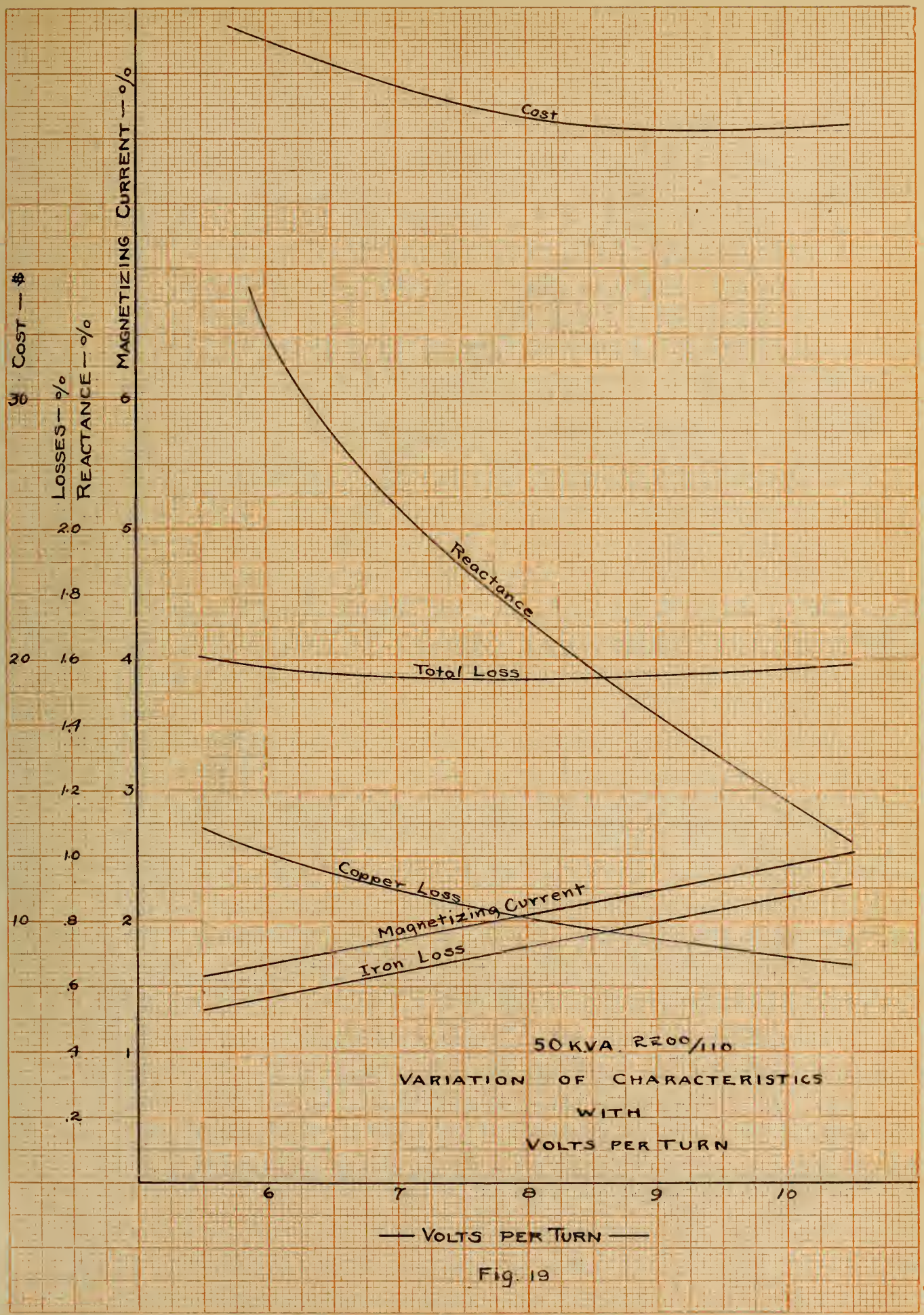

Cost — $
Losses — %
Reactance — %
Magnetizing Current — %
30
20
10
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
6
5
4
3
2
1
Cost
Reactance
Total Loss
Copper Loss
Magnetizing Current
Iron Loss
50 KVA. 2200/110
VARIATION OF CHARACTERISTICS
WITH
VOLTS PER TURN
6
7
8
9
10
— VOLTS PER TURN —

Fig. 19

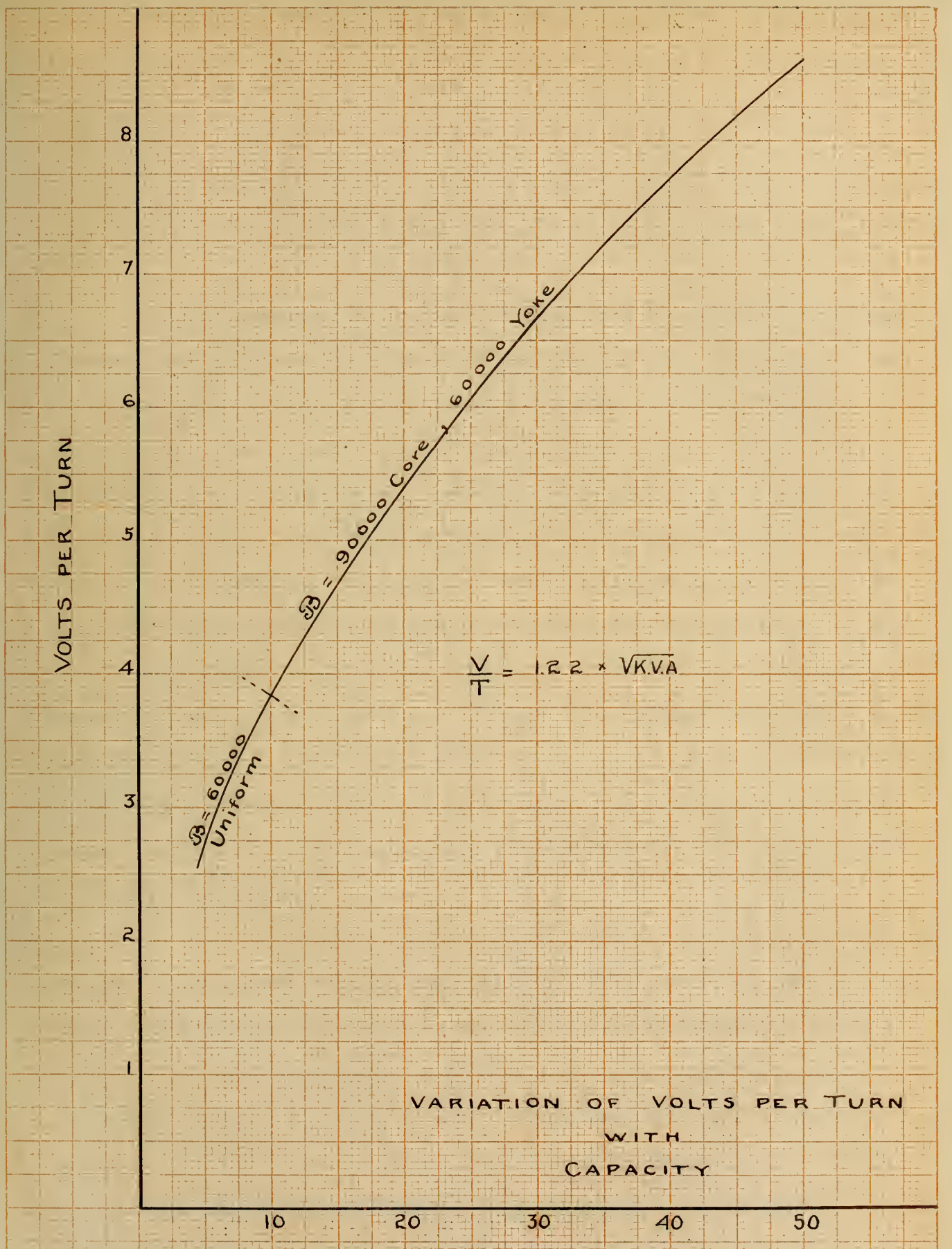

VOLTS PER TURN
8
7
6
5
4
3
2
1
$\mathfrak{B}$ = 90000 Core + 60000 Yoke
$\mathfrak{B}$ = 60000 Uniform
$\frac{V}{T} = 1.22 \times \sqrt{K.V.A}$
VARIATION OF VOLTS PER TURN
WITH
CAPACITY
10
20
30
40
50
K.V.A. CAPACITY

Fig. 20

CONCLUSION.

It was necessary in this work to assume the properties of iron used. A poorer grade of material would, of course, give a greater iron loss and magnetizing current. Applying this effect to the volts per turn curves, it is found that for equal losses the volts per turn would be decreased. Roughly, an increase of 20 per cent in the iron loss reduces the volts per turn 5 per cent to 8 per cent. The corresponding increase in the magnetizing current would not be prohibitive in the larger sizes, say from 15 K. V. A. up. For the smaller sizes the remedy would be to use the lower core-density.

The section of the copper conductor as used is also an arbitrary assumption based on good practice. The cost of the transformer could be decreased by using a greater current-density, but such a change could only be justified by balancing the decrease in the cost of capital against the annual expense incurred by the increased loss. The heating of the transformer would, of course, be increased.

It is of interest to note the variation of the losses, cost, reactance, and magnetizing current with the capacity. For this purpose, curves, Fig. 21, have been plotted, giving these quantities as ordinates against K. V. A. capacity. The cost-ordinate in these curves, as in all the curves shown, is understood to represent merely the cost of copper and iron with no labor charges. The cost of construction is practically independent of the design as regards volts per turn, and depends upon the size of the transformer. No practical data could be obtained regarding the actual construction cost.

The above curves, Fig. 21, indicate and calculation verifies that the magnetizing current will reach an objectionable value for the small sizes below 10 K. V. A., hence the use of uniform density, say 60,000, is advisable. If the above formula be applied to the design of these small transformers with uniform density, the losses will not be equal, but the general design will be very good.

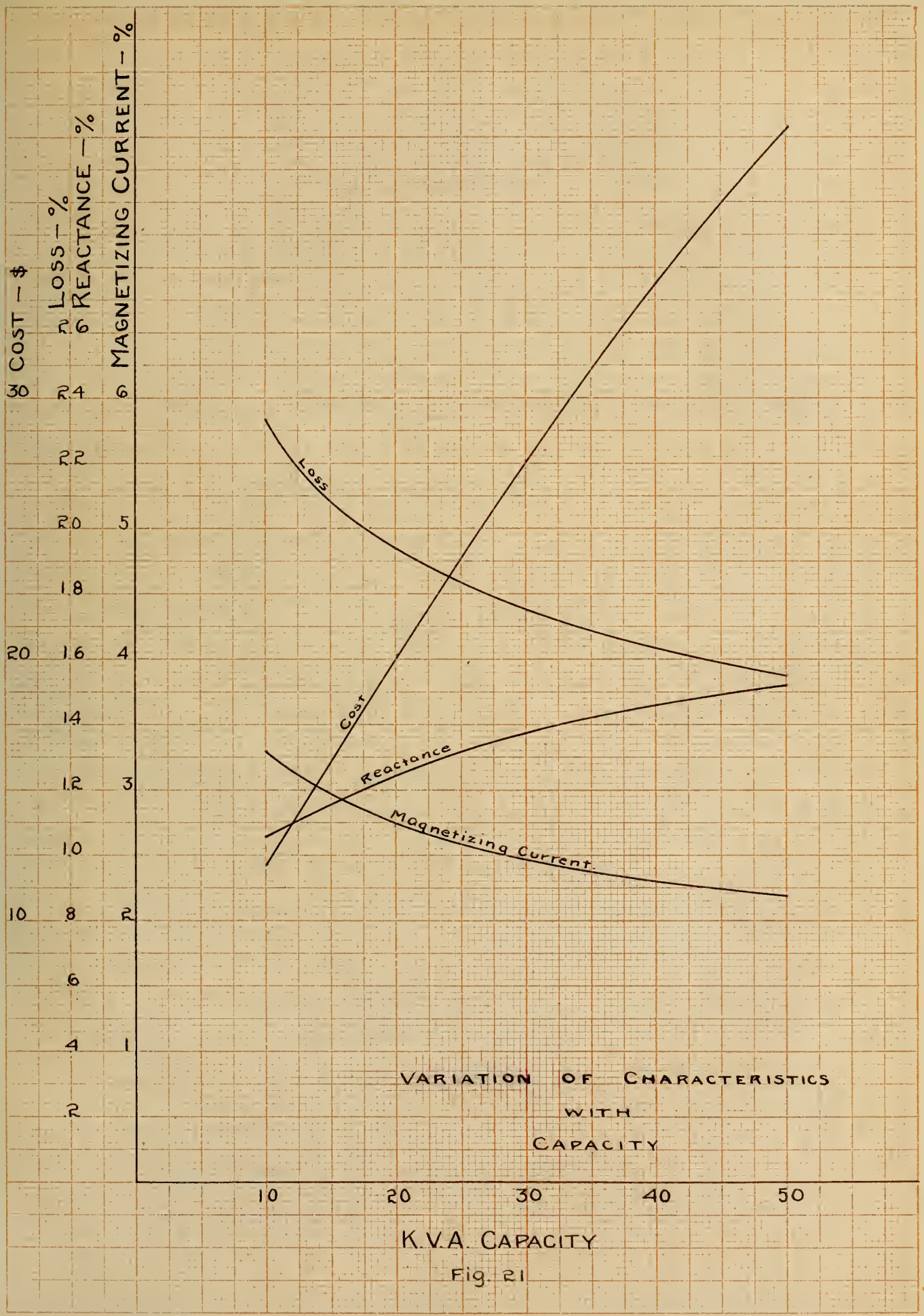

COST — $
LOSS — %
REACTANCE — %
MAGNETIZING CURRENT — %
30
20
10
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
6
5
4
3
2
1
Loss
Cost
Reactance
Magnetizing Current
VARIATION OF CHARACTERISTICS
WITH
CAPACITY
10
20
30
40
50
K.V.A. CAPACITY

Fig. 21

SAMPLE CALCULATIONS

In order to verify this formula, and also to show the general method used in the calculation, a 5 K. V. A., 2200/110 volt transformer design is included below, using the volts per turn as determined from the formula.

5 K. V. A. capacity, 2200/110 volt, 60-cycle, shell type transformer.

Following the above suggestion, the density is uniform, 60,000 core and yoke.

$V/T = 1.22\sqrt{K.\ V.\ A.} = 1.22\sqrt{5} = 2.73$

$\text{Turns} = \frac{2200}{2.73} = 800$ approximately

$\text{Amp.} = \frac{5000}{2200} = 2.27$ amp.

$\text{Section of Conductor} = \frac{2.27}{1000} = .00227$ sq. in.

$$\phi = \frac{2200 \times 10^8}{4.44 \times 60 \times 800} = 1,030,000 \text{ lines}$$

$\text{Iron Area} = \frac{1,030,000}{60,000} = 17.2$ sq. in.

$\text{Core Area} = \frac{17.2}{.9} = 19.1$ sq. in.

Core Section (ratio 2.25:1) = 2.91 in. X 6.56 in.

Copper Area. Primary must equal Secondary

Copper Section Pri. = 800 X .00227 = 1.82 sq. in.

Copper Section Sec. = 1.82 sq. in.

$\text{Coil Section Pri.} = \frac{1.82}{.57} = 3.19$ sq. in.

$\text{Coil Section Sec.} = \frac{1.82}{.8} = 2.27$ sq. in.

Coil Section Total = 3.19 + 2.27 = 6.46 sq. in.

Insulation = 1.08 sq. in.

Total Window Area = 6.46 + 1.08 = 7.54 sq. in.

Window Section (ratio 2:1) = 1.94 in. X 3.88 in.

$$X' = \frac{2.27 \times 2.54}{(3.88 \quad .12) \times 2} = .765 \text{ cm.}$$

$$Y' = \frac{3.19 \times 2.54}{(3.88 \quad .12) \times 2} = 1.08 \text{ cm.}$$

$$3\,g = .06 \times 2.54 \times 3 = .457 \text{ cm.}$$

$$X' + Y' + 3\,g = 2.3 \text{ cm.}$$

$$(N)^2 = 640{,}000$$

$$\lambda = 13.12 \text{ in.}$$

$$\ell = 3.88 \text{ in.}$$

$$L = \frac{4\pi (N)^2 \lambda}{1 \times 3 \times 10^9}[X + Y + 3\,g] \times 1.15 \quad 0.239 \text{ ohms}$$

$$X \text{ per cent} = \frac{2\pi f LI}{2200} = 2\pi \times \frac{60 \times .0239 \times 2.27}{2200} = .93 \text{ per cent}$$

$$\text{Mean Turn} = 13.12 + (1.94 + 2.91)\pi = 28.33 \text{ in.}$$

$$\text{Length Pri. Wire} = \frac{28.33}{12} \times 8000 = 1890 \text{ ft.}$$

$$\text{Res.} = \frac{.00000947 \times 1890}{.00227} = 7.87 \text{ ohms}$$

Since mean turn is the same for Primary and Secondary,

$$\text{Equivalent Res. Pri.} = 2 \times 7.87 = 15.74 \text{ ohms}$$

$$I^2R \text{ per cent} = \frac{15.74 \times \overline{2.27}^2 \times 100}{5000} = 1.62 \text{ per cent}$$

$$\text{Volume Copper} \quad \frac{2 \times 1890 \times .00227}{144} = .0596 \text{ cu. ft.}$$

$$\text{Wt.} = .0596 \times 550 = 32.8 \text{ lb.}$$

Cost at \$.16 = \$5.26

Volume Iron = .193 cu. ft.

Wt. Iron = 460 X .193 = 88.8 lb.

Cost Iron at \$.04 = \$3.55

$$\text{Iron Loss per cent} \quad \frac{88.8 \times .82 \times 100}{5000} = 1.46 \text{ per cent}$$

Total Cost $5.26 + 3.55 = $8.81

Mmf = 17.46 X 3 X .707 = 30 A. T.

$$I_m \text{ per cent} = \frac{30 \times 100}{800 \times 2.27} = 1.65 \text{ per cent}$$

The magnetizing current is calculated by determining the mean path through the iron, as shown in Fig. 1 then, knowing the density, the ampere turns are easily calculated by the usual method.

These results show that while the I^2R loss does not exactly equal the iron loss they are close together and at the same time the magnetizing current and reactance are kept down so that the transformer is of very satisfactory design.

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